Engineering Notes

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Aerodynamic Requirements at the Indianapolis Motor Speedway

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Introduction

IN the present Note, we consider the aerodynamic downforce generation necessary to achieve specified lap times at the Indianapolis Motor Speedway (IMS). Aerodynamic downloads can be created by the use of airfoils, winglets, wings, and the shape of the racing car itself. As Fig. 1 shows, an increase in normal loading on a tire produces a higher lateral cornering force for a given slip angle setting. Aerodynamic downforce provides an increase in tire normal loading and thus higher friction without increasing the mass and inertia of the racing car itself.

IMS consists of a 2.5-mile (4.0-km) oval formed of two long and two short straightaways, and interconnected by four turns, as shown in Fig. 2. In this paper, a model is developed to predict lap times and speeds by explicitly modeling vehicle cornering performance, including the effects of aerodynamic downloading. Quantification of the model is based on measurement of a limited number of instantaneous speeds.

Observation of driving technique indicates that each turn is initiated well before the physical start of the turn, as shown in Fig. 3. Measurements indicate that turns 1 and 3 are begun about 300-350 ft (92-107 m) before the track actually begins to curve. In effect such a maneuver makes turns 1 and 2 into a nearly continuous arced path, rather than two distinct turns. Turns 3 and 4 are similar. The nominal length of each long straightaway is 3300 ft (1006 m). By assuming that the actual entry and exit points to and from each long straightaway are 330 ft (101 m) before and after each turn, the effective length of each long straightaway is 2640 ft (805 m). (See Figs. 2 and 3.)

Computation of Velocities and Required Downforce

The author acted as a member of the timing and scoring crew for the 1983 Indianapolis 500-Mile Race qualifications. Instantaneous velocities were measured during each qualification attempt at three locations around the racetrack equipped with radar timing devices; these are shown on Fig. 2. During qualifications only one racing car was on the track during any given attempt. These readings were taken for the 33 cars that qualified for the 1983 Indianapolis 500-Mile Race. Readings 2, 3, and 4 are accurate to within about 0.2% error; reading 1 is more inaccurate because of the position of the timing personnel and the resultant cosine effect described by Fisher.[1]

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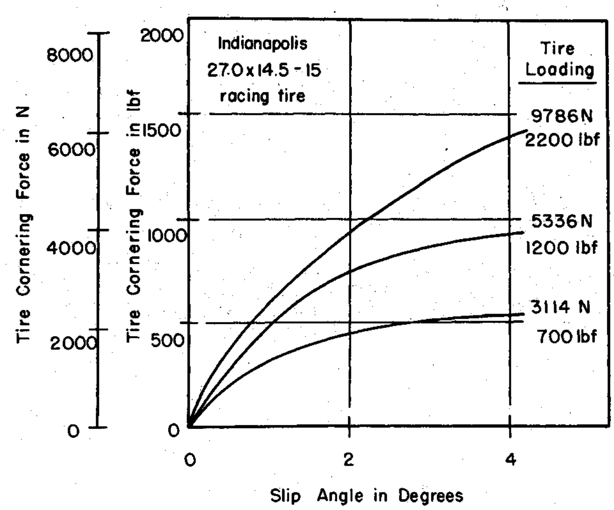
To model intermediate velocities, IMS was divided into 20-ft (6-m) sections. At racing speeds, the time required to traverse one section is about 0.075 s, so this level of discretization is an excellent approximation to a continuous model. Next, bank angle and arc radius for each discretized section were determined. The maximum bank angle of the track surface at the apex of each turn is 9.2 deg. Bank angles were linearly interpolated from this maximum value forward and backward to the physical entrance and exit of each of the four turns. The straightaway bank angle is zero.

A fifth-order equation was used to approximate the radius of the racing trajectory through each turn, as described in Ref. 2:

$$r_i = r_{\min} + (10{,}000 - r_{\min})(x_i/\mathrm{dist})^5 \tag{1}$$

where r_i is the local radius, constant at segment i; $r_{\min}$ is the minimum radius, equal to one-half car width plus the turn radius at the inside edge of each turn, measured at its apex; 10,000 is the representation of a track radius of infinity; dist stands for the curvilinear distance along the track from the apex of each turn to a point at which the car has exited the turn; and x_i is the distance from the turn apex to the ith track segment.

The maximum width of Indianapolis racing cars is 80 in (203 cm) (Ref. 3); the IMS is 60 ft (18 m) wide at the apex of each turn; finally, the radius of each turn to the centerline of the racetrack is 840 ft (256 m). The minimum radius to the vehicle center of gravity at the apex of each turn is thus $840-30+80/2\times 12=813.3$ ft. The ratio x_i/dist is zero at the turn apex and 1 at the actual beginning or end of each turn. Turns were also assumed to be symmetrical, in the sense that



Nondimensional Maximum Forces

Tire Loading		Lateral Force/Tire Load
3114 N	700 lbf	1.13
5336 N	1200 lbf	1.01
9786N	2200 lbf	0.98

Fig. 1 Tire performance characteristics.

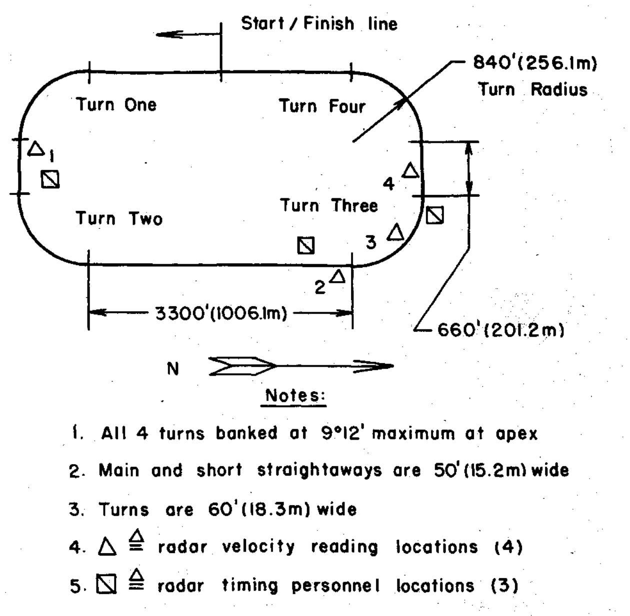


Fig. 2 IMS layout.

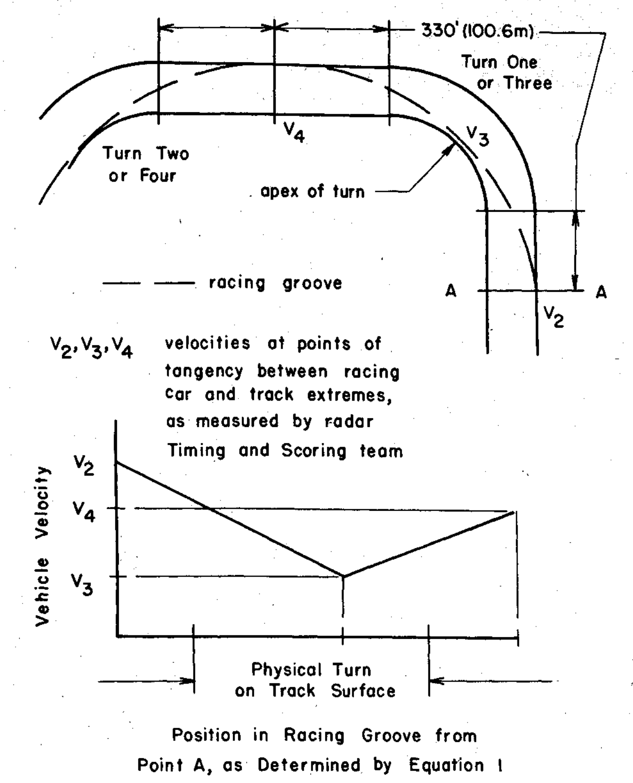


Fig. 3 Interpolation of velocity in turns for a sample car.

Eq. (1) describes only one-half of each turning maneuver (either onset of turning to apex, or apex to completion of turn). Aerial inspection of the tire marks left by racing shows this to be a valid assumption. Lastly, each turn was assumed to be identical to all others; differences were assumed to be second order and were neglected.

From radar timing data taken at locations 2, 3, and 4, interpolated velocity values were obtained for the first half and second half of each turn. The velocity distribution throughout each of the two long straightaways was also obtained. Velocity increases upon exciting either turn 2 or 4, but reaches a maximum about halfway down each straightaway. At terminal velocity, propulsive power equals drag power consumption

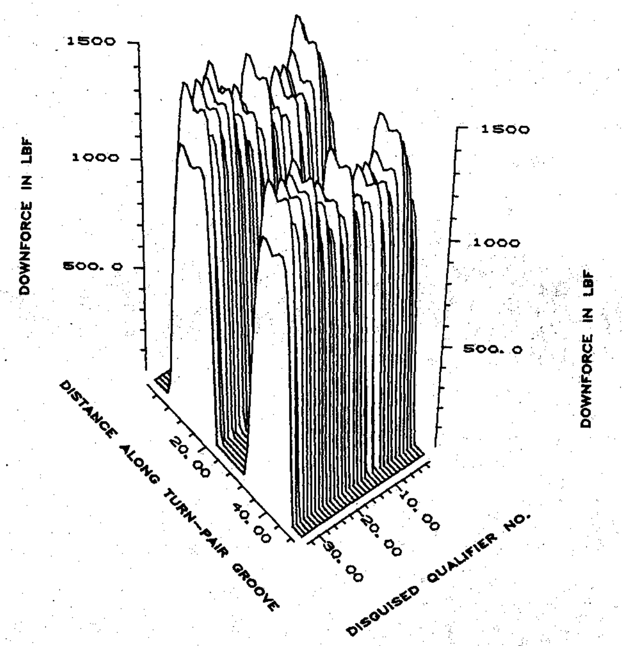


Fig. 4 Downforce requirements vs position in turn for 33 1983 qualifying vehicles; coefficient of friction = 1.6.

and further acceleration cannot occur. To model this performance, a first-order exponential fit was used:

$$V_i = K_0 + K_1(1 - e^{-x_i/y}) \tag{2}$$

where v_i is the velocity at the ith segment as measured from the beginning of the straightaway; k_0 is the velocity at beginning of each long straightaway; k_1 is the difference between k_0 and terminal velocity (terminal velocity equals radar reading 2); x_i is the distance from the beginning of the straightaway to the ith segment; and y is a parameter, nominal value equals 264.

Large but reasonable variations in the parameter y produce total straightaway times that exhibit little variation.

Comparison of Computed and Actual Lap Times

Velocities were computed for each 20-ft (6-m) segment of the racetrack. Euler integration was used to compute the time required for one lap using these velocities. This lap time was multiplied by 4 and compared with the total qualification time recorded by the 33 1983 qualifiers; the comparison is shown in Table 1. Average velocities were also computed for each qualifier, and these are compared in Table 1 with actual velocity averages achieved during qualification attempts. Note that the percent errors for time and velocity are not identical. This is due to local variations in velocity at each track segment from the assumed velocity profile.

Aerodynamic Downforce Requirements

Aerodynamic downforce varies as a complex function of vehicle velocity, and is difficult to compute. However, if one assumes that each driver extracts maximum cornering performance at all times, it is possible to compute the downforce required at each combination of radius, bank angle, and forward velocity:

$$F_c \cos\theta - F_f - F_w \sin\theta = 0 \tag{3}$$

where F_f is the tire friction force μx (tire normal force); F_c is the lateral centrifugal force at local radius r_i; F_w is the vehicle weight force; and F_a is the aerodynamic downforce developed through all methods (wings, ground effects, etc.).

Table 1 Comparison of estimated and actual lap times and velocities for 33 1983 qualifying vehicles

Disguised qualifier number	Official four lap time, s	Computed four lap time, s	Time based, % error	Official four lap speed, mph	Computed four lap speed, mph	Speed based, % error
Q1	188.292	184.798	−1.856	191.192	194.807	1.891
Q2	184.678	182.606	−1.122	194.934	197.146	1.135
Q3	180.014	178.156	−1.032	199.984	202.070	1.043
Q4	177.936	175.364	−1.445	202.320	205.287	1.467
Q5	178.635	175.261	−1.889	201.528	205.408	1.925
Q6	182.133	178.161	−2.181	197.658	202.064	2.229
Q7	183.008	180.396	−1.427	196.713	199.560	1.448
Q8	181.246	178.947	−1.268	198.625	201.177	1.285
Q9	180.400	176.857	−1.964	199.557	203.555	2.003
Q10	181.272	178.769	−1.381	198.597	201.377	1.400
Q11	199.976	194.797	−2.590	180.022	184.808	2.658
Q12	181.252	179.126	−1.173	198.618	200.976	1.187
Q13	196.565	193.686	−1.465	183.146	185.868	1.486
Q14	180.227	178.563	−0.923	199.748	201.609	0.932
Q15	181.601	179.474	−1.171	198.237	200.586	1.185
Q16	180.538	178.486	−1.137	199.404	201.696	1.150
Q17	178.213	175.615	−1.458	202.005	204.994	1.479
Q18	183.982	181.432	−1.386	195.671	198.421	1.406
Q19	182.179	180.795	−0.760	197.608	199.120	0.765
Q20	180.637	178.014	−1.452	199.295	202.231	1.473
Q21	184.513	181.720	−1.514	195.108	198.107	1.537
Q22	182.043	180.175	−1.026	197.755	199.806	1.037
Q23	180.096	177.886	−1.227	199.893	202.377	1.243
Q24	181.320	179.780	−0.849	198.544	200.244	0.856
Q25	179.903	177.670	−1.241	200.108	202.623	1.257
Q26	173.582	172.125	−0.839	207.395	209.150	0.846
Q27	176.742	174.874	−1.057	203.687	205.862	1.068
Q28	178.258	176.461	−1.008	201.954	204.011	1.018
Q29	176.211	174.253	−1.111	204.301	206.596	1.123
Q30	178.089	176.194	−1.064	202.146	204.320	1.075
Q31	186.535	183.120	−1.831	192.993	196.592	1.865
Q32	175.292	173.194	−1.197	205.372	207.859	1.211
Q33	183.729	181.505	−1.211	195.941	198.342	1.225

If $F_f = \mu x$ (the total normal force on the tires), then

$$F_c \cos\theta - \mu(F_a + F_w \cos\theta) - F_w \sin\theta = 0 \tag{4}$$

and the necessary downforce is then

$$F_a = \frac{(mv^2/r)\cos\theta - F_w \sin\theta - \mu F_w \cos\theta}{\mu} \tag{5}$$

where m is the vehicle mass. A coefficient of friction between tire and track of 1.6 is assumed. Changes in this coefficient will affect results.

The downforce requirements for each of the 33 1983 qualifiers are shown in Fig. 4 as a function of turn location. The downforce required in each of the long straightaways was assumed to be zero because no turning maneuver is being executed.

Conclusions

Downforce requirements for racing cars at IMS can be determined using the above techniques. Simulation results for both velocity and time show errors that are consistently in the 1-2% range, with many below 1%.

References

[1]Fisher, D.P., "Shortcomings of Radar Speed Measurements," *IEEE Spectrum,* Dec. 1980, pp. 28-31.

[2]"Computer Simulation of Watkins Glen Grand Prix Circuit Performance," Vehicle Research Department, Cornell Aeronautical Laboratory, Buffalo, N.Y., CAL Rept. ZL-500-K-1, New York, 1971.

[3]Technical Rules Bulletin, United States Auto Club, 4910 West 16th Street, Speedway, Indiana.

Slow and Fast State Variables for Three-Dimensional Flight Dynamics

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Introduction

BECAUSE of the complexity of the sixth-order point-mass equations of aircraft motion, there long has been interest in using reduced-order models in aircraft flight mechanics. This interest has increased greatly because of the development of singular perturbation methods for aircraft trajectory optimization.[1,2] The success of these methods depends essentially on identifying appropriate slow and fast state variables.

It is well known[2] that state variables altitude (h) and velocity (V) are approximately the same speed for supersonic aircraft and are, therefore, not time-scale-separable for

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